Finite Elements, Design Optimization, and Nondestructive Evaluation: A Review in Magnetics, and Future Directions in GPU-based, Element-by-Element Coupled Optimization and NDE

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Abstract:
Most important journal papers in magnetics are selected from conference records with quick review and subject to stringent page limits. The literature as a result is unsatisfactory, inadequately attributing previous works and without sufficient details to replicate work presented. This paper therefore reviews mathematical optimization in synthesis and nondestructive evaluation (NDE) by the finite element method in magnetics. The review identifies the earliest papers. Thereafter this paper proposes and establishes the feasibility of coupled problem optimization using the genetic algorithm to avoid mesh induced minima which hurt gradient based methods. The genetic algorithm, while avoiding the need for derivatives, results in having to undertake even more numerous finite element solutions. Although the genetic algorithm has been applied in optimization, in coupled systems the number of object function evaluations doubles. We there examine the use of graphics processing units (GPUs) to handle the immense computational load. GPUs have recently been introduced in finite element analysis but their memory limits are often not recognized and are critically limiting when parallelizing the several solutions required in optimization. To overcome this limit, element-by-element finite element matrix processing is employed, making coupled problems practicable on GPUs. We overcome the memory limits faced by others.

1. Inverse Problems for Design – A Review

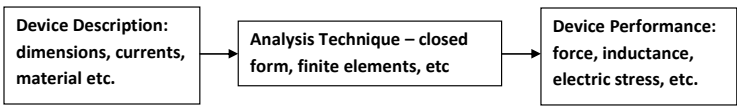

Figure 1: The Typical Forward Problem: Analysis

The direct problem with which the finite element method started [1-3] has a device governed by a particular differential equation, say the Poisson equation

$$-\epsilon\nabla^2\varphi = \rho \tag{1}$$

as was solved by the finite element method by Zienkiewicz and Cheung in their classic paper [3]. Once we have φ – which may be electric potential, pressure, magnetic vector pressure etc. depending on the system – we may compute performance descriptions like inductance, force, etc. (Fig. 1). That is, from the system description, we compute performance. This is analysis.

The inverse problem – the more practically realistic problem, which is synthesis – goes from the right hand side of Fig. 1 to its left. That is, wanting a performance, computing the system description from it. Thus the computational design assignment may be this: compute the size and other descriptions

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of a motor that can produce so much torque. In industry this was done by the cycle of design-make-test-redesign. This required an expert to redesign and took long. In time by the 1970s instead of making and testing, we analyzed, solving the direct problem by the finite element method.

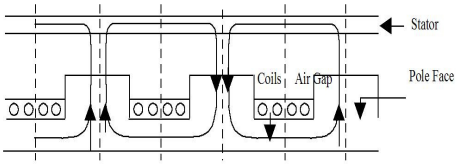


Figure 2: Pole-Faces to be Shaped

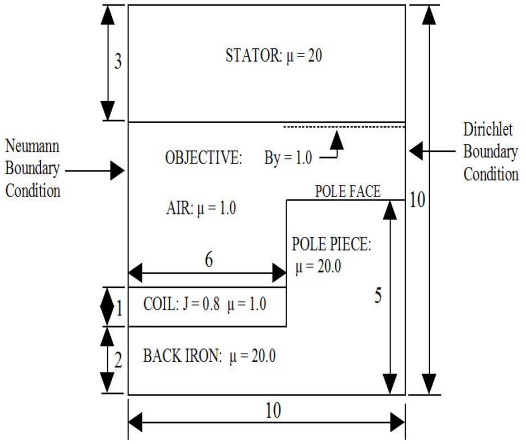


Figure 3: Minimal Problem using Symmetry

It was left to engineers dealing with stress analysis and fluids to couple optimization with the finite element method [5-7], and the second half of the 1970s and 1980s would be the time for true synthesis – solving for geometric shape and material values from design criteria. The earliest persons to automate this cycle in magnetics were Marrocco and Pironneau in 1978 [6]. They attempted to optimize the shape of the magnetic pole of a recording head so that the fringing effect at the edges of a pole could be countered so as to realize the object of constant flux density B in a the recording head. (Fig. 2 gives a similar problem with a repeating alternating pole system from electric machinery where a constant flux density is required on top of each pole to facilitate alternating waveform generation. The minimal boundary value problem for analysis is shown in Fig. 3).

Marrocco and Pironneau [6] located their work in the latter's 1976 doctoral thesis at Université Pierre-et-Marie-Curie (also known as UPMC and Paris VI), optimizing structural and fluid systems. That is, their work may be seen as parallel to the 1976 work of Arora and Hang [7] who established finite element optimization in a journal. They approached this problem by defining an object function F consisting of the square of the difference between the computed and desired flux densities. Thus the problem is one of optimizing – i.e., minimizing – F which is a function of parameters $p_1, p_2, \cdots$ defining the geometry and which are computed so as to minimize F.

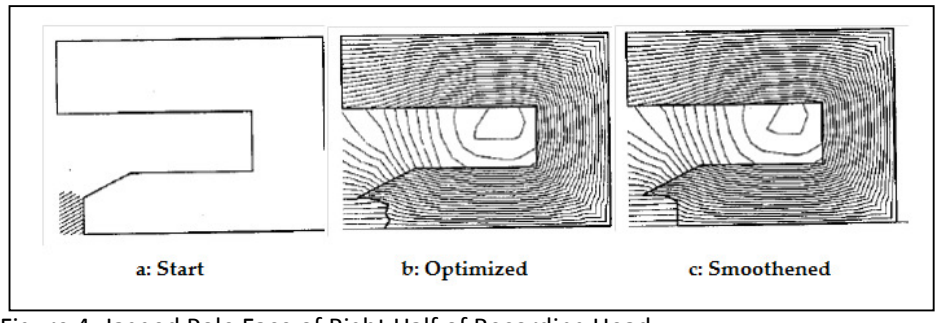


Figure 4: Jagged Pole Face of Right Half of Recording Head

Their results are shown in Fig. 4. The nonsmooth jagged contour in Fig. 4b that they realized is practically not a manufacturable shape. This they addressed by smoothening the pole face as in Fig. 4c. However, Marrocco and Pironneau, aware of the problem of jagged contours, have further addressed it to permit smoothening by allowing nodes to move only along prescribed paths.

Although the comprehensive 1984 book by Pironneau on optimization [8] deals with the theory of imposing constraints and applies them to many systems, the results presented from magnetics are the same as presented by Marrocco and Pironneau much earlier without constraints. Pironneau, a widely experienced pioneer scientist in optimization and finite element analysis, particularly in fluids, was not focused on magnetics. He with Marrocco solved this problem to demonstrate the broad applicability of their methods of finite element optimization and then moved on. The line of work broached by Pironneau would remain untapped for a while until another French group would pick it up using computational optimization just coming to the fore then [9].

In magnetics the Ecole Nationale Supérieure d'Ingénieurs Electriciens de Grenoble (ENSIEG) group led by J.C. Sabonnadiere, J.L. Coulomb and G. Meunier would bring mathematical optimization to bear on finite element analysis design in 1989 [9]. The ENSIEG group, however, ignored the 1978 magnetics paper by Marrocco and Pironneau [6] and sought inspiration from the seminal 1976 structural optimization paper by Arora and Hang [7].

This early work flowing from the ENSIEG group used gradients based methods, steepest descent in particular [5]. Here the change in parameters of device description {p} is against the gradient of the object function F because in one-dimensional analogy the minimum point is to the right of locations with negative gradient and to the left of those with positive gradient:

$$\{p\} = \{p\} - \alpha \frac{\partial F}{\partial \{p\}} \tag{2}$$

where the amount of change α is determined by a line search [5]. The computation of the gradient ∇F $(i.e., \partial F/\partial\{p\})$ was previously by finite difference, computing F through a finite element solution corresponding to a given {p} and then in turn changing each component p_i by an infinitesimal amount and re-computing F to get $\partial F/\partial p_i \approx \delta F/\delta p_i$. Thus the component of ∇F at each iterative step with n components of {p} took n+1 finite element solutions and then once the direction of change of {p},$-\nabla F$, is established several more finite element solutions were need to be sought during the line search as α in (2) is progressively increased and the problem iteratively solved until the minimum of F in that direction is identified [5]. Each changed {p} means a new geometry and therefore a new mesh. For a seamless iterative process, automatic mesh generators are required that can yield a mesh corresponding to a given {p}.

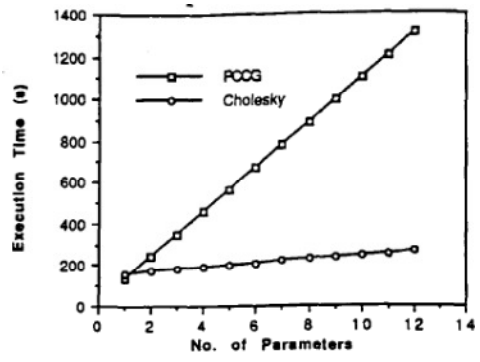


Figure 5: Slow Rise in Solution Time by Cholesky's LU Method as Parameters go up

The finite difference computation of the gradient of F had been known to be notoriously inaccurate from force computations by the virtual work principle [10]. However Coulomb [11-13] of the ENSIEG group identified a one-step solution for the computation of $\nabla F$from the finite element solution without resort to a second solution for determining the change in F with change in each component of {p}. This discovery occurred while working with the virtual work principle for force computation in magnetics where the force F in the direction s is computed by differentiating the stored magnetostatic energy $\mathcal{U}_m$: $F = -\partial \mathcal{U}_m/\partial s$. The finite element approximation of the Poisson equation (1) leads to the matrix equation

$$[P]\{\varphi\} = \{Q\} \tag{3}$$

where the Dirichlet matrix [P] is usually evaluated numerically from element coordinates [14, 15]. Prior to numerical evaluation however, [P] is an explicit function of the nodal coordinates and therefore differentiable explicitly. That is, $\partial[P]/\partial s$ for force computation or $\partial[P]/\partial p_i$ in designing parameters p_i may be numerically evaluated after differentiating the explicit form of [P] as a symbolic expression in terms of element coordinates after mapping nodal coordinates to s or p_i as appropriate [15, 16]. $\partial\{\varphi\}/\partial p_i$ may thereupon be computed from, upon differentiating (3):

$$[P]\frac{d\{\varphi\}}{dp_i} = \frac{d\{Q\}}{dp_i} - \frac{d[P]}{dp_i}\{\varphi\} \tag{4}$$

Although the Incomplete Cholesky Conjugate Gradients (ICCG) method is usually the preferred method of solving matrix equations with sparse symmetric positive definite coefficient matrices as from the finite element method, in this particular case it is far more efficient to use the Cholesky factorization method as seen in Fig. 5 [14, 17]. In Cholesky's method most of the work is in decomposing [P] into its lower and upper triangular Cholesky factors [L] and [U] after which all that is left to do is the quick forward elimination and back-substitution. In this instance, since equation (3) for $\{\varphi\}$ and the many equations (4) for $\partial\{\varphi\}/\partial p_i$ share the same coefficient matrix [P], once the Cholesky factors are computed in solving (3) for $\{\varphi\}$, they may be used to solve (4) for $\partial\{\varphi\}/\partial p_i$ with trivial extra work [17]. That is why in Fig. 5 as the number of p's goes up, the solution time for Cholesky's scheme remains practically flat, taking up time only for the forward elimination and back-substitution.

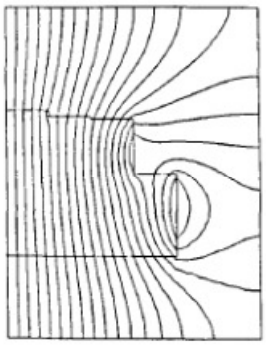


Figure 6: Jagged Optimized Shapes

This seminal ENSIEG work on force computation through differentiation [11-13] led by 1989 to the next logical step of applying derivative information for gradients based optimization [9, 18]. Although the Grenoble group too worked with constraints, perhaps to ensure basic requirements like lengths not going negative, they did not apply constraints to ensure the smoothness of shapes. For example, Fig. 6 from [9] presents a pole shape they synthesized for a linear change in the horizontal direction of the vertical flux density, where the jagged contour is evident.

2. Other Early Papers on Optimization in Magnetics

This seminal Grenoble paper in 1989 [9] following Marrocco and Pironneau's original work [6], both from France, opened up the subject of optimization in magnetics. This new subject innovation had two aspects: one – the key – the idea of the inverse problem posed through the minimization of an object function, and, two, the method of optimization to find the minimum of that object function.

Each magnetics paper that followed, for the most part added a new aspect in terms of method of optimization while failing to acknowledge the preceding foundational work on posing the inverse problem through object functions. Unfortunately the *IEEE Transactions on Magnetics*, *Journal of Applied Physics* and a few others with their policy of quick turn-around through selecting most of their papers from conferences, had a flurry of papers that did not reference these seminal works by Marroco and Pironneau [6] or the ENSIEG group [9]. Because of the route and methods of paper selection described in detail in section 4, these indexed-journal papers often applied a new method of mathematical optimization and effectively purported to bring inverse problem solution into

magnetics; whereas they were really bringing into magnetics new ways of minimizing the object function introduced to magnetics by Marroco and Pironneau, and ENSIEG. The page limit with conference issues of journals did not permit space for replicability so that these papers failed to allow others to repeat their work easily and build upon it; the journals were focused on results rather than method because of such stringent page limits.

The next two papers on finite element optimization in magnetics after the ENSIEG paper came early in 1990, from Germany, both selected conference papers carried in the same issue of the *IEEE Transactions on Magnetics* [19, 20]. Russenschuck's newness [19] was in the methods of optimization, for example on the Rosenbrock and SUMT optimization algorithms for searching for the object function's minimum by gradient methods [5]. No new information was offered on how the gradient ∇F of the object function was calculated. Seminal sources on gradient computation were not referenced. Neither did they acknowledge the French works. Nor did Schafer-Jotter and Muller [20] who gave more details and introduced the zeroth order, statistical-based simulated annealing method to magnetics; however they do not seem to have recognized the advantages of not having to compute gradients.

Likewise, a few months later in 1990, an Austrian group, also ignoring the French work, did not mention the problem of jagged shapes but avoided the problem by modeling shapes by fourth, sixth and eighth order polynomials and optimizing for the polynomial coefficients [21]. However, it was puzzling because it is well known that high order polynomials in modeling the curve of Reluctivity ν Versus B^2 for steels yield undulating (squiggly) shapes and this problem is avoided by, just as in finite element shape functions, several short shapes of low order (cubics at most) with the values and slopes matched at the boundaries [22]. Presumably the results they presented are for third order polynomials because they lack the undulations to be expected. For when we tried high order models we did get highly undulating shape profiles. However, that paper usefully introduced to magnetics the evolution strategy (a variant of the genetic algorithm brought into magnetics by a Korean group under Hahn to optimize a coil gun [23]. The solution there was not by finite elements but by circuit models).

A Japanese group under Nakata [24], also as early as 1991 and not referencing the French works but the later work of Schafer-Jotter and Muller [20], optimized a magnetic circuit by minimizing a least-square object function by Rosenbrock's search method [5]. An early Italian paper from Jan. 1992 [25] had very powerful results in 3-D but offered little information in terms of method or the object function except to say they used pattern search with constraints. If they had given more details it would have been seminal for having branched into optimization in 3-D magnetics.

A British group [26] brought in simulated annealing without mentioning the seminal French work nor the work with simulated annealing of Schafer-Jotter and Muller [20]. Their geometric changes involved no continuous change of shape but square bits that were switched on and off as finite element optimization in magnetics proceeded to grow as a field. A Korean group did not mention constraints but came up with results of smooth shapes using the steepest descent method [27]. Other papers introduced parallel computation on shared memory systems for finite element optimization [15, 17, 28].

3. Constraints, Specialized Meshes

Although several papers had been presented on gradients based optimization, none had mentioned the problem of jagged contours and that of artificial mesh-induced local minima that are not intrinsic to the physical problem. As noted, the jagged contour results were smoothened by Marrocco and Pironneau [6, 8] and the ENSIEG group paid no attention to the jagged contours of Fig. 6. Others had used polynomial fits of surfaces thereby avoiding the problem [21].

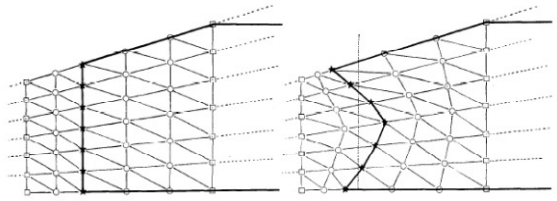


Figure 7: Unwitting use of Elastic Mesh by Marrocco and Pirnonneau [6]

No one among the authors of the preceding papers at the time had recognized the problem of mesh induced local minima in object functions. As a geometry changes and an element violates the Delaunay criterion for meshes [14], the nodal connections are changed by the mesh generator thereby making the evaluated object function undergo a C^1 discontinuity. The local minima encountered by researchers (according to numerous personal communications at the time such as with the authors of [25]) were thought to be systemic (i.e., inherent to the problem being solved) and bypassed by restarting the iterations from another point.

Recognizing this problem Subramaniam *et al.* [29] suggest a tunneling function and an assortment of algorithms so that when one fails another may be started to identify the minimum. However, the real cause of the local minima in the object functions, would soon be identified as owing to changes in nodal connections as the shape evolved under synthesis [30, 31]. Two solutions were offered: either keep the nodal connections fixed so that as geometric changes occur the meshes are elastically pulled or crunched, or, alternatively, use a zeroth order search method so that the local mesh-induced minima are not a problem. In light of this knowledge, when Marrocco and Pironneau's paper [6] was examined, it turned out that they had, working in an era where there were no automatic mesh generators, used an elastically distorted mesh with the connections not changing as shown in Fig. 7. This had given them smooth object functions without their recognition that the specific mesh changes they used had made them avoid the attendant problem of mesh-induced local minima. A special mesh generator elastically to deform the mesh while keeping nodal connections fixed was created by Krishnakumar for his doctoral thesis [32].

To address jagged contours a few successful approaches have been proposed and employed. Explaining using the pole face described by n parameter heights $p_1, p_2, \ldots, p_n$ as shown in Fig. 8, the object function is defined by

$$F(\{p\}) = \frac{1}{2}\sum_{i=1}^{9}\left(B_i^y - 1\right)^2 \tag{5}$$

where B_i^y is the vertical component of flux density at the measuring points i which are also shown in Fig. 6. B_i^y at the 9 measuring points would be forced to the value 1 T when F goes down to its lowest value 0. Those were times – the 1980s – when shape optimization was just taking off. The mathematical optimization formulation used by Pironneau in his book was complete. He implemented

$$Minimize\ F = F(\{p\} \tag{6a}$$

subject to the equality constraints

$$H_i(\{p\}) = 0 \quad i = 1, 2, \ldots k \tag{6b}$$

and inequality constraints

$$G_i(\{p\}) \leq 0 \quad i = 1,2, \ldots l \tag{6c}$$

Inequality constraints of the type $J_i\{p\} \geq 0$ may be recast as in (6c): $G_i(\{p\}) = -J_i(\{p\}) \leq 0$. Those of the form $a \leq J_i\{p\} \leq b$ as $G_i = (J_i - a)(J_i - b) \leq 0$. This form with upper and lower limits is often normalized as $G_i = (J_i - a)(J_i - b)/(b - a) \leq 0$).

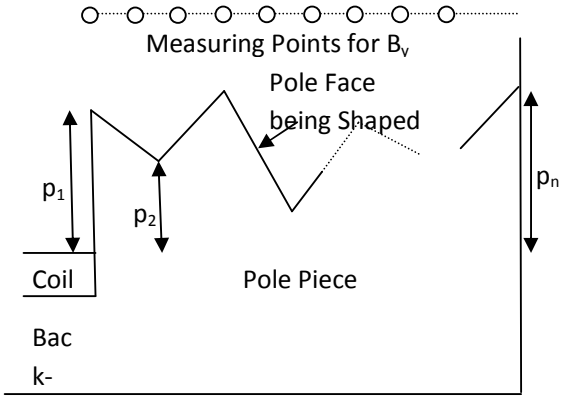


Figure 8: Parameterized Pole Face Description Vector {p}

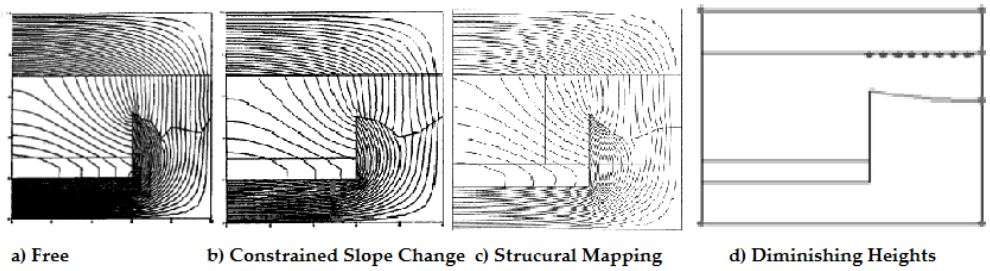


Figure 9: Smoothening Shape by Constraints

As already mentioned Marrocco and Pironneau [6] did not use constraints in magnetics in 1978 or 1984 to enforce the smoothness of the pole-face. Nor did the ENSIEG group [9]. With constraints it has been shown that jagged contours result as in Fig. 4b and Fig. 9a. But with constraints forcing the straight line segments from point i-1 to point i to be within say a certain angle of the slope from point i to point i+1,

$$\left|\frac{p_{i-1}-p_i}{x_{i-1}-x_i} - \frac{p_i-p_{i+1}}{x_i-x_{i+1}}\right| \leq \epsilon \tag{7}$$

shaped surfaces would be of the form seen in Fig. 9b [33]. The result can be relatively smooth but undulating and could be objectionable to those called to manufacture a recording head like this, although it meets the design object perfectly. Using the multiple solutions there exist for this problem, another approach regards

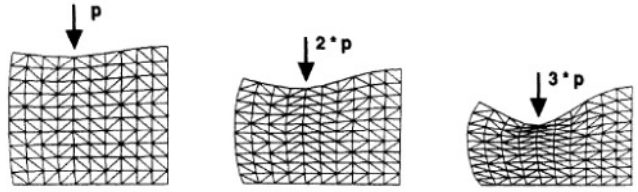


Figure 10: Elastic Mesh from Deformation under Pin-Point Load

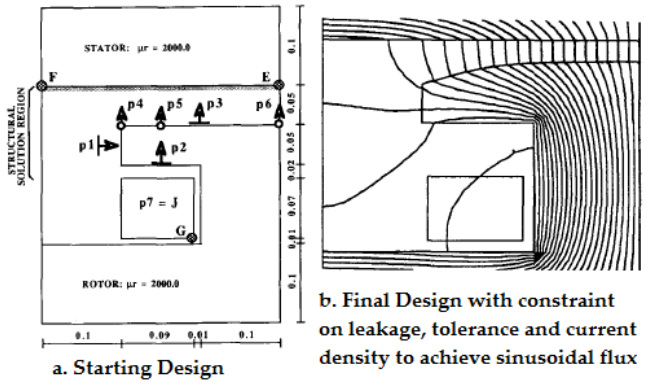


Figure 11: Structurally Deformed Alternator Optimization

the recording head artificially as made up of a rubber like substance so that if pressed down by a pin-point force it would deform smoothly as in Figs. 9d and 10 [31]. Thus the question is recast as "What is the set of pin-point forces F_{pp} which when applied to the surface, deform it to a form that would yield a uniform flux density – i.e, minimize the object function F. The artificially assigned mechanical properties of the material ensure a smooth profile of the shaped geometry as shown in Fig. 11. Thus it is a two part problem whose importance is in laying the mathematical groundwork for coupled field problems [34]. A structural problem solves for {p} from pin-point forces $\{F_{pp}\}$

$$[S]\{p\} = \{F_{pp}\} \tag{8}$$

and the magnetics problem gives the magnetic vector potential $\{A\}$ sourced by current density J:

$$[P]\{A\} = \{J\} \tag{9}$$

Since optimization requires gradients with respect to the parameters of design, in this case the pin-point forces, we need

$$\frac{dF}{dF_{pp}} = \frac{\partial F}{\partial F_{pp}} + \frac{\partial F}{\partial \{A\}}\frac{d\{A\}}{dF_{pp}} \tag{10}$$

$\partial F/\partial F_{pp}$ accounts for explicit appearances of F_{pp} in F and is nearly always zero. $\partial F/\partial A$ is easily obtained because the object function is defined in terms of the flux density B and the curl of A is B. We now compute $d\{A\}/dF_{pp}$, where {A} and F_{pp} are in the electrical and structural systems, from

$$\frac{d\{A\}}{dF_{pp}} = \frac{d\{A\}}{d\{p\}}\frac{d\{p\}}{dF_{pp}} \tag{11}$$

The two derivatives on the right hand side are obtained by differentiating and solving the equations (8) and (9) for the derivatives. Alternatively we can work without forces but some part of {p} replaced by forced displacements. Fig. 11 from Weeber's doctoral work [31, 35] shows the designed alternator to yield a sinusoidal stator flux distribution.

A third option is to work with constraints on the heights p_i numbered from the left to right of the form

$$p_1 \geq p_2 \geq p_3 \geq \cdots p_n \tag{12}$$

This will yield a solution as in Fig. 9d, again giving an object function going down to 0 because of the multiplicity of solutions possible.

Because this problem of shaping a pole to yield a constant flux density has numerous solutions it has now come to be a standard benchmark problem for testing software purporting to optimize geometries [36].

Weeber and Hoole [37], recognizing that much of the work in magnetics is reinventing the wheel already invented under structural analysis and that there was a wealth of information already in that literature unknown to the magnetics community in electrical engineering, brought several methods

from structural and civil engineering into electromagnetics. Thus the important subregion method was brought into magnetics and the part where the shape is being optimized made a subregion so that computations were greatly simplified [38].

4. Quality of Magnetics Papers and Special Via-Conference Journal Issues

Papers on numerical methods are difficult to evaluate. Unlike with old explicit solutions where to accept an argument one simply follows the authors' derivations, with numerical methods a reviewer for proper assessment needs to program the method again repeating work which might have been done over a doctoral thesis. This cannot practicably be done.

Recognizing this, thanks to several conferences on magnetics having arrangements for selected papers to be quickly carried by journals, many important results have seen the light of day instead of having to languish for long periods for review. But as a result, as we have seen in the previous review of papers in this important area of magnetics, past work was not properly attributed and results needing a lot more development were published without providing sufficient detail to replicate them.

By far the vast majority of papers in computational electromagnetics appear in the *IEEE Transactions on Magnetics* and the *Journal of Applied Physics* via the IEEE Conference on Electromagnetic Field Computation (for which the first author was once in charge in various capacities), and the COMPUMAG, Intermag, and Magnetism and Magnetic Materials Conferences. Regular papers with leisurely review and no page limits are comparatively few in these journals. However, respectability of results has become an issue as a result. The papers being author-prepared for quick turnaround, poorly typed papers in bad grammar are not unknown. In the days of ribbon printers such little attention was given by the IEEE to printing quality in the rush to meet deadlines, that some papers given in readable quality for printing in indexed transactions were totally unreadable because of photo-reproducing without proper exposure [39].

As a conference is planned, a publication date is set and a maximum of 2 months is given for review for quick turnaround of a few hundred papers. As a result, whereas with regular journals a paper is carefully matched to reviewers and when reviewers say they are not competent new reviewers are found; on this accelerated route, reviewers are set well before a paper is submitted and the tight schedule does not allow reviewers to be easily changed (although that is occasionally done). A great disservice appears to have been done to engineering science by this rush. Thus for an issue of one *IEEE Transactions on Magnetics* the first author was asked to review 3 papers over the 4-day duration of the conference in Tokyo without access to his library or facilities for internet searches. Whatever he read was in between sessions and dinners with friends after the day's proceedings. While that may be an extreme example, reviews are generally inadequate. For example even when two weeks were given for review at the reviewer's usual office, when the first author of this paper was the Guest Editor for selecting papers from such a conference, a paper presenting methodology and equations based on the finite element method had meshes and results from the boundary element method. In the hurry the 2 reviewers had accepted the paper! (It was, of course, rejected).

If a reviewer asks for a major change, even a good paper is effectively rejected for lack of time. So reviewers are reluctant to ask for major changes. If a section needs elaboration for repeatability of

results, reviewers are again reluctant to ask for that because what can be said in the 3 or 4 pages allowed the author? This is how in magnetics a lot of partial work and papers without attribution to previous works have made it into indexed journals. In the worst oddity, a paper rejected for the *Journal of Applied Physics* from the MMM Conference, still appears as a one-page summary in the journal (because it was presented), giving indexed journal respectability to a rejected paper. An example is cited as reference [40].

Getting a paper is thus a matter of funds for travel and registration plus the multiple-paper surcharges at conferences, and then playing the lottery where the more papers one throws into the conference, the better the chance of something getting accepted. Typically it costs $3000 to get a paper by this route (for conference registration, hotel and subsistence, and travel). As a result unlike in the old days these journals give better access to those who can pay, while the quality of papers suffers. The professional societies standing behind these prestigious conferences make a lot of money.

When these issues were raised with the IEEE, the response was that poor authors may submit to the regular issues. But the continued free access to regular papers is poor compensation given the 2-plus years that regular papers can take and the higher quality that regular papers need for acceptance. These journals have argued that there is no loss of quality because the rate of acceptance for regular and via-conference papers is similar. That is a fallacious argument because authors are very careful about what they submit to a regular journal but going the conference route they sometimes submit several papers knowing the acceptance rate will guarantee a proportion to be accepted. It is like comparing the products of 1) a university that carefully admits 100 students and graduates 90% and 2) another university admitting 10,000 students and graduating 90%. The products cannot be of the same quality even though both have a 90% retention rate.

A new dimension of the problem as journals attempt reform (recognizing the problems without admitting to them) is from their imposing a limit on the number of pages of each special conference issue. Previously there could be any number of pages so long as the conference paid for them. Now, in a context where nearly all reviewers are attendees at the conference, if a reviewer accepts a paper, it could be a vote against his or her own paper!

Until these problems are addressed papers in magnetics will continue to suffer in quality.

5. Prospects for Further Extensions

5.1 Extensions

In magnetics a lot of important work has been done in device shape optimization since 1978. The purpose of this section is to establish future directions for harnessing the power of these methods.

5.2 Applications to NDE: Assessment of Severity of Interior Defects

Hoole, working with the ENSIEG group on a Summer assignment just after their seminal work in 1989, would use the fact that the ENSIEG methods could be equally applied to non-destructive evaluation (NDE) [16]. That is, where design synthesis computed the design vector {p} to match electromagnetic fields corresponding to design goals (which are the performance specifications), in NDE the interior defect is described by the design vector {p} and the performance is the externally

measured fields. That is we find the defect that would match exterior measurements through a least-square object function expressing the difference between the measurements and the fields computed in the presence of the defect. Thereby the shape and location of the defect are identified.

These methods have since been carried into applications such as testing for cracks in oil pipelines, underground petrol storage tanks, nuclear reactors, etc. [41-44]. An external current coil is taken over the structure subject to testing (Fig. 12). Knowing the response field when the structure is defect free, a change in measured field is used to flag a defect.

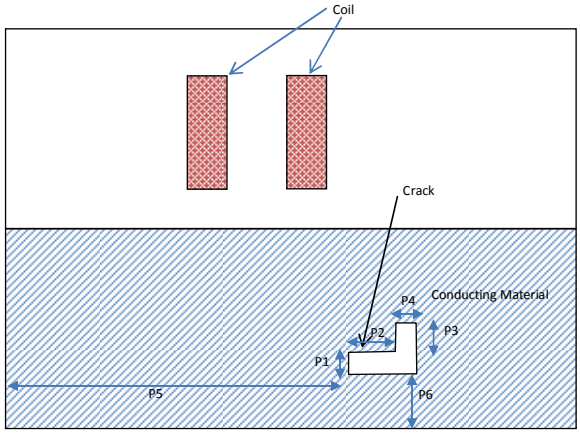
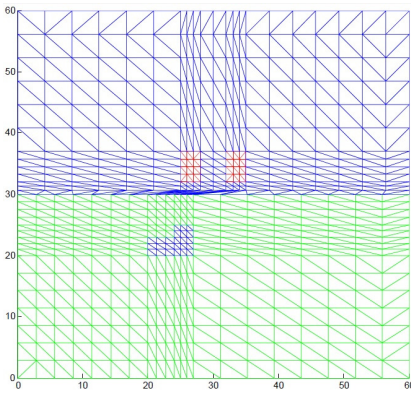
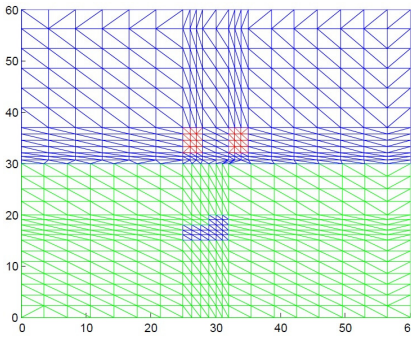


Figure 12: Problem-Specific Parametric Mesh Generator

However, when there is an inaccessible defect, it is important to know the severity of the problem. For army ground vehicles hulls with minor rusting setting in, the current testing methods would flag a defect and the vehicle withdrawn from deployment. But so withdrawing it without proper assessment of the defect may be an unwarranted waste. That is, the nature of the defect has to be assessed to avoid wasteful withdrawal from service. Having the response waveform from measurement, we postulate a defect defined by dimensional parameters p_i as in Fig. 12 Not knowing the actual shape, the more parameters we have the better for accurate assessment of defect.



a. Initial Mesh



b. Elastically Deformed Final Mesh

Figure 13: Mesh Generator (Limited Shape)

To establish feasibility for this method, we need a special mesh generator modeling the crack defined by parametric location and shape. We created such a mesh generator for establishing feasibility as shown in Figs. 12 and 13b, created a crack and computed the fields along measuring points outside the steel plate. As the dimensions {p} of Fig. 12 change during optimization, the mesh is crunched and pulled from the starting mesh of Fig. 13a to the final design of Fig. 13b. This mesh is therefore suitable for testing gradient methods as well as zeroth order methods. But it needs further generalization to model all manner of defects.

Taking the results as the "measured field," we had to "discover" the shape and location of this crack as described by parameters {p}. The shape was identified by the genetic algorithm as well as simulated annealing (with the former working better as discussed below). The final

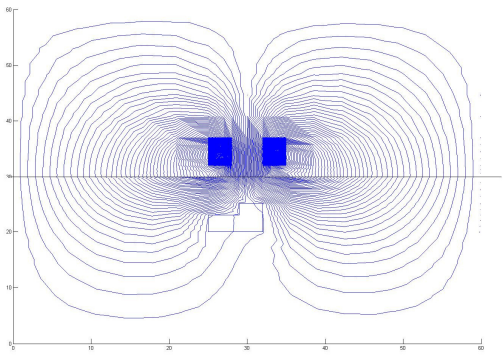


Figure 14: Crack Recreated from Exterior Fields using 10 parameters

defect is seen in Fig 13b. and the magnetic fields in Fig. 14. Ultimately this problem would need to be done in 3-D where the computational load would be high because each component of the 3-component magnetic vector potential would be complex [14].

Our experience is that gradient techniques are fast in computation but slow to set up because of the programming time to have special mesh generators. Going by the literature Preis *et al.* [45], staunch advocates of the zeroth order evolution strategy, merely say it is competitive with its higher order deterministic counterparts (which we take to mean the same in time at best), but claim its "robustness and generality" are superior. This we agree with because search methods will never see mesh-induced artificial local minima as a problem. In contrast to what they say, Simkin and Trowbridge [26] aver that simulated annealing and the evolution strategy take many more function evolutions. This is also our experience and we would add that the genetic algorithm works faster than simulated annealing. Haupt [46] advises that the genetic algorithm is best for many discrete parameters and the gradient methods for where there are but a few continuous parameters. We have gone up to 30 continuous parameters using gradient methods without problems. There seemed good reasons to go either way.

For now therefore in this feasibility study, a zeroth order method like the genetic algorithm or simulated annealing would be best. We therefore decided to test both methods, simulated annealing and genetic algorithm, both zeroth order methods where no derivative calculations are required. The least square object function F was defined as the square of the difference between the exterior fields we need to get and those computed from the postulated crack. The fitness function $f = 1/(1 + F)$ converged to almost 1. The genetic algorithm worked faster than simulated annealing as seen from Tables 1 and 2. For the genetic algorithm, a comparable object function is computed from $F = (1 - f)/f$. It is seen that the genetic algorithm reaches a comparable object function value much faster than simulated annealing.

Table 1: Performance of Genetic Algorithm

Population Size	Error (%)	f Fitness Function	F, Object Function	Time (s)
10	6.7	0.94×10^{-4}	1067.0	2.00
20	5.46	0.0018	554.55	3.88
50	2.09	0.9974	0.002	9.65

Table 2: Performance of Simulated Annealing

Iterations	F, Object Function	Time (s)
500	0.0448	14.25
1000	0.0146	28.22
4000	0.0282	119.12
40000	0.0075	1144.42

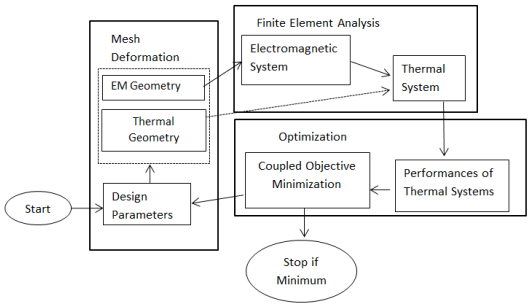


Figure 15: Procedure for Coupled Electrothermal Optimization

5.3 Coupled Problem Shape Optimization

Coupled field problems are where one field system with the shapes to be optimized influences another field system in which the objects of design are defined. A good example is electro-heating [47]. The electrical system provides the joule system that has to be shaped so as to produce a particular heat distribution. Two example industry applications are metal forming where molten metal is heated through heavy currents to produce the forces to make the molten metal subject to the extrusion or turning forces we want [48, 49]. The application of the ENSIEG force computation methods to this problem is obvious. A second example is hyperthermia treatment for oncology where exterior electrodes attempt to burn interior cancerous tissue [50].

The procedure for coupled problem optimization is summarized in Fig. 15. As noted we have already solved this by gradient techniques and chain rule differentiation [34] using the same procedure as used for deforming the finite element mesh under pin point forces [31, 35].

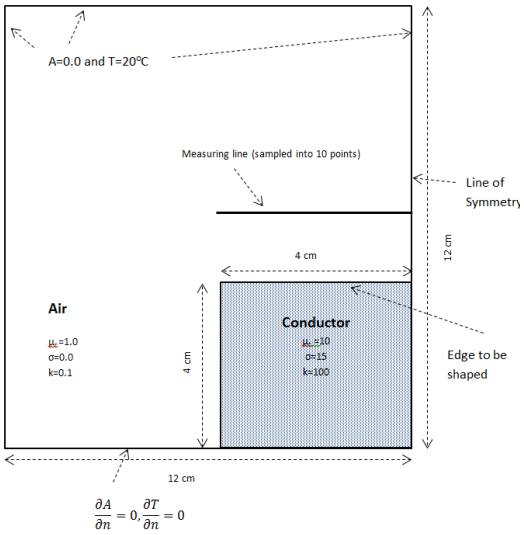


Figure 16: The Boundary Value Problem

In comparing methods of optimization generally by far the gradient based methods are the fastest as already noted. However as also noted they are difficult to program and to set up because of the requirements on elastic meshes to avoid mesh-induced local minima which are fatal to gradient optimization. But we have successfully optimized the shape of a conductor so that just above it along a straight line the temperature is constant – corresponding to the coupled boundary values problem of Fig. 16 showing a quarter of the system where the current in the conductor, subject to eddy effects [14], heats it thereby producing a temperature profile around it. We wish to have a constant temperature along the line of measuring points shown above the conductor. The object function then is the sum of the squares of the difference between computed temperature T({p}) and the desired constant temperature at the measuring points. To this end of optimizing the shape we have already computed the gradients using chain-rule differentiation using special mesh generators that avoid mesh-induced minima [30, 33]. In our experience, for coupled problems gradient based methods are all the more difficult because the meshes for the two field systems are often different and the programming very problem specific and tough. Another method is therefore required without any requirement for gradients – that is the genetic algorithm which has been found to be satisfactory except for needing numerous matrix solutions. In coupled problems this computational load, however, will be twice as much.

5.4 GPU in Coupled Problem Optimization

We wish to resort to the genetic algorithm but it involves many function calls – that is many new meshes and a finite element solution for each. One way to reduce the computational load is to resort to parallel computation on shared memory machines [15, 17]. However, shared memory machines are expensive and usually come with 8 or 16 processors (because of technical difficulties in sharing the memory between processors) so that parallelization is limited.

Recently using the graphics processing unit (GPU) on PCs has been proposed and implemented for finite element solution [51]. The GPUs are cheap and by default come with every PC. They allow multiple launches of a computational kernel on many GPU strings in parallel.

However, what is not readily recognized – for example not mentioned in [46] – in the computational literature although stated in hardware manuals is that there is a GPU memory limit, presently at 4 GB. For us the limit is real and has been recently encountered in finite element analysis [52, 53]. When we try to process several finite element solutions simultaneously in parallel we do hit the limit. To test the limits we ran a problem with ever increasing size. The results are presented in Table 3. The limit was reached around a matrix size of 10,000 by 10,000.

Table 3: Hitting the 4 GB Limit at Matrix Size 10^4x10^4

Size	Storage (in MB)		
	Normal	Profile	Sparse
100	0.04005	0.00443	0.00652
400	0.06866	0.04127	0.01686
900	3.10707	0.12714	0.03632
1600	9.79614	0.28701	0.07027
2500	23.88954	0.54378	0.11368
6000	137.44354	1.54289	0.27344
8000	244.29321	2.66143	0.35942
10000	381.66046	4.08210	0.45021

This limit is too small for us when several such matrix equations have to be processed simultaneously on different strings on a GPU. When sparse and profile storage schemes are employed [14] much bigger matrix sizes are possible – for example with the more efficient sparse storage we have processed matrices of size close to 10^8x10^8 before reaching 4 GB as in Table 4 based on Table 3 and additional computations. Curve fitting was employed to project some of the information in Table 4 because only the sparse storage scheme goes up to 4 GB, necessitating the other storage schemes to be theoretical at matrix sizes which are not feasible within 4 GB. These are huge matrices.

Table 4: Projected Memory

Matrix Size	Regular (MB)	Profile (in MB)	Sparse (MB)
20000	1525.2	14.7265	0.90
30000	3430.5	32.1285	1.35
50000	9526.5	87.0594	2.25
100000	38097.0	341.7935	4.51
500000	9.5224e+05	8417.7	22.58
1000000	3.8089e+06	33608	45.17
5000000	9.5220e+07	8.390e+05	225.85
10000000	3.8088e+08	3.35e+06	451.70
50000000	9.5220e+09	8.39e+07	2258.50
100000000	3.8088e+10	3.35e+08	4517.00

Nonetheless in optimization when we launch several strings in parallel even the efficient sparse storage scheme can be limiting and we must seek out new methods of handling this storage and workload. However, we note that the use of the sparse scheme of storage necessitates an iterative method without matrix decomposition; for matrix decomposition would require profile storage.

Kiss *et al.* [52] too have run into memory limits but at a smaller matrix size of 3,836,282 while solving a single matrix equation (as opposed to our 10^8x10^8). In their via-conference IEEE Transactions paper few details are given to permit following. Dziekonski *et al.* [53] as we have noted have bypassed the problem by using multiple GPU clusters which are not commonly available and it is technically difficult for most finite element analysts who would not be competent dealing with hardware arrangements on their own. So large coupled problems with two field systems to be solved, for all practical purposes, are beyond the capabilities of GPU memory. In fact we found that we quickly exceeded the memory limit and our program crashed.

To overcome this, we used a method of the early 1980s when, working with the then new IBM PC 286 machine, we had a memory limit of 612 kB which could not hold even a trivial matrix in memory. What we used to do [54] was employ the Jacobi methods of matrix solution (power systems engineers call it Gauss-Seidel) in a modified form. For example in solving (3), [P]{φ}={Q}, the Gauss-Seidel iterations commonly used by power engineers, is an improvement on the older Gauss iterations. In Gauss-Seidel in each iteration m+1 we use the latest available values of the unknowns φ, using equation i of (3) to compute φ_i treating only φ_i as the unknown and all the other variables as known and given by their latest values in the iteration cycle:

$$\varphi_i^{m+1} = \frac{1}{P_{ii}}\left\{\sum_{k=1}^{i-1} P_{ik}\varphi_k^{m+1} + \sum_{ki+1}^{n} P_{ik}\varphi_k^{m}\right\} \tag{13}$$

with obvious modifications for i =1 and i = n. In this algorithm φ_{i-1} must be computed before φ_i. Here at iteration m+1, computing φ_i in the order i=1 to n, φ is at values of iteration m+1 up to the (i-1)th component of { φ } and at the value of the previous iteration m for values after i. It is therefore necessarily a sequential algorithm. The older displaced Gauss iterations uses the old iteration's value for computing all φ_i in iteration m+1. Therefore the computation of a particular $\varphi_i$value is independent of the computation of all other φ_i values and therefore parallelizable:

$$\varphi_i^{m+1} = \frac{1}{P_{ii}}\left\{Q_i - \sum_{k=1}^{i-1} P_{ik}\varphi_k^{m} - \sum_{k=i+1}^{n} P_{ik}\varphi_k^{m}\right\} \tag{14}$$

This is inefficient in the context of sequential computations. But in this case of parallelization as several strings on the GPU, it is highly efficient. But the problem of memory needs to be addressed as each string must carry [P]. We may address this by not forming the matrix [P]. If [D] is the matrix [P] with all off diagonal elements eliminated, then Gauss's iterations in this modified form gives,

$$[D]\{A\}^{m+1} = \{Q\} - [P - D]\{A\}^{m} \tag{15}$$

Thus without forming [P], the operation of the right hand side of (15) can be effected by taking each finite element in turn, computing the local 3x3 Dirichlet matrix $[P]^L$ and using that because

$$[P] = \sum_{Elements}[P]^L \tag{16}$$

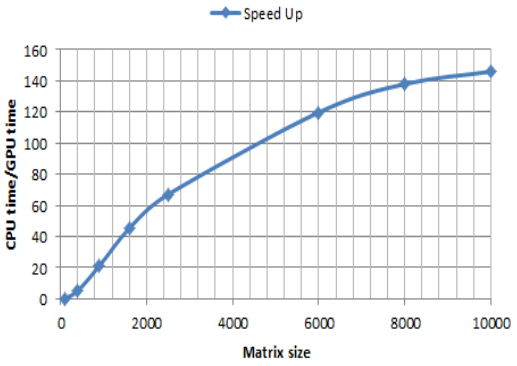


Figure 17: Speedup by Element-by-Element Iterations

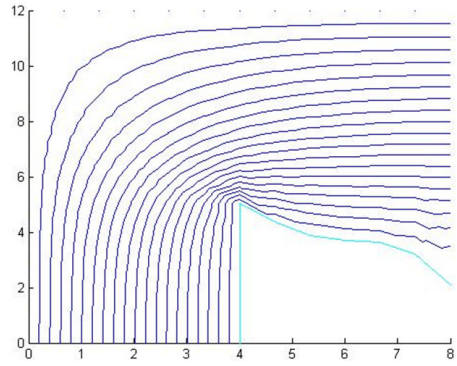


Figure 18: Genetic Algorithm Shaping for Design Goal

So as each $[P]^L$ is formed in each string, the three values of $\{\varphi\}^m$ may be taken and subtracted as in the right hand side of (14). We tested this and the results are shown in Fig. 17. We were able to go up to matrix sizes 100,000x100,000, speed-up computation not being possible for higher sizes with normal storage. We project being able to go well-beyond to 100,000,000x100,000,000 without hitting memory limits.

We then applied the same idea to the more efficient Incomplete Cholesky Preconditioned Conjugate gradients (ICCG) method [14] as laid out by Mahinthakumar and Hoole [55] and Carey *et al.* [56] for shared memory systems with very similar speedup of about 130 – impossible on a shared memory system where with 16 processors speedup will be below 15, accounting for one processor for coordination of the other 15 and time for exchange of information between processors

In this work, the Incomplete Cholesky Conjugate Gradients matrix solver was parallelized on the GPU and we observed a speed-up of 146.351 for the matrix size 10,000x10,000 (Fig. 17). The shaped conductor is shown in Fig. 18. In the GA kernel, implemented here there was no internal parallelization – that is no parallelization of the matrix computation routines within the genetic algorithm such as of matrix computations. This could have been done for even greater gain.

In some ways element-by-element work this is like what Kiss *et al*.[52] have done but only with ICCG. While they applied it to the solution of one matrix equation, we have solved an optimization problem on a GPU. We have applied that to Gauss-Seidel with multiple matrix equation solutions for optimization. Whereas Dziekonski *et al.* [53] have encountered memory limits working with a single matrix equation of 3,836,282 degrees of freedom by their element by element method, we are able to go up to 100,000,000 degrees of freedom without hitting any limit. At least for what was compared, we can run 100/3.862 or about 26 parallel optimizations streams for problems as big as that, and numerous times more for smaller everyday problems.

This number 26 is well above the number of parameters being usually optimized for designs so that such approaches are feasible for genetic algorithm parallelization on GPUs.

Conclusions

This paper has reviewed finite element optimization in magnetics and its natural extension to nondestructive evaluation. Much good work has been done in the subject area of magnetics but because of the culture of the journals in which this work is published, attribution to previous works is poor and the emphasis is on results rather than repeatable methodology. Perhaps with good but misguided intentions of fast dissemination of results, the subject area of finite element field computation in magnetics appears to have invited an insidious culture of quick and easy publication that is harmful to the subject. In the long term this will be seen as invidious by colleagues from other areas of science and engineering who compete for the same awards, recognitions, and promotions.

The review of the work in finite element optimization in magnetics has identified the original papers as coming from French scientists. This paper has pointed out future directions of this subject in coupled field problem optimization with the genetic algorithm, the development of mesh generators based on parametric description and the use of GPUs to accelerate the process through modified algorithms. The mesh generator developed conforms to the requirements for smooth object functions so that future work can compare zeroth and first order methods in magnetics.

The genetic algorithm used in optimization requires numerous function evaluations. In coupled problems this is doubled. So the use of GPUs is critical to parallelize the computations and speed them up. But that also doubles the memory requirements on GPUs which suffer severe memory limits. Others have used element-by-element processing for matrix solution processing to reduce memory loads. But they too have run up against these memory limits. To overcome these, the use of clusters of GPUs has been reported but this is not satisfactory because of their not being widely available. We have been able to solve much larger problems than reported on a single GPU using sparse storage, without recourse to clusters by element-by-element processing with Jacobi's method which takes less memory than ICCG..

Significant speed up of 146 has been shown by using element-by-element iterative approaches to finite element matrix solution. The memory savings achieved without multicore systems offers promise for running several strings of genetic algorithm kernels without hitting memory limits. It is also possible to parallelize each string into several parts parallelizing the matrix solution scheme in a kernel because of the availability of memory freed up by the element-by-element approach. Some preliminary work has been presented to establish the feasibility of these suggestions with 26 design variables.

Future directions must engage more general mesh generators to model any shape of crack and three dimensional analysis of eddy current fields to locate and characterize interior defects.

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